Introduction

The Key Stage 3 National Strategy is based on four important principles:

- **Expectations**: establishing high expectations for all pupils and setting challenging targets for them to achieve;
- **Progression**: strengthening the transition from Key Stage 2 to Key Stage 3 and ensuring progression in teaching and learning across Key Stage 3;
- **Engagement**: promoting approaches to teaching and learning that engage and motivate pupils and demand their active participation;
- **Transformation**: strengthening teaching and learning through a programme of professional development and practical support.

This guidance focuses on how these principles apply to the teaching and learning of mathematics for pupils learning English as an additional language (EAL). It suggests some strategies to help teachers provide access for pupils at different points of learning English. It is important to support them to develop both their understanding and use of the English language and to enhance their learning in mathematics lessons.

The Strategy has high expectations for all pupils, and the inclusion of pupils learning English as an additional language is a fundamental principle. This is highlighted in the Framework for teaching mathematics: Years 7, 8 and 9 (section 1, page 35):

> It is all too easy to underestimate what pupils can do mathematically simply because they are new learners of the English language. The expectation should be that they progress in their mathematical learning at the same rate as other pupils of their age.

The Ofsted report Managing support for the attainment of pupils from minority ethnic groups (October 2001) identifies factors that enable bilingual learners to develop their English successfully:

- joint planning between mainstream and specialist ethnic minority achievement (EMA) staff;
- a focus on the content of the lesson, ensuring appropriate cognitive challenge;
- a parallel focus on the language necessary to complete the task;
- activities that enable pupils to rehearse and explore the language they need;
- opportunities to use and build on their first-language skills, where appropriate;
- continuing support with writing through, for example, the use of matrices for organising information and writing frames for more extended contributions.

Acquisition of academic language can take considerably longer to develop than social language. This advanced level of proficiency in the language for learning is crucial to the attainment of pupils for whom English is an additional language in all subjects of the curriculum.

The report draws attention to the ‘considerable evidence that once proficiency in English was achieved, the progress for pupils with EAL across the curriculum was rapid and their attainment on a par with or higher than that of their monolingual peers’.
Securing progress for pupils learning English as an additional language: the role of the subject leader

Success for pupils learning EAL depends on close monitoring of their academic and personal targets. Meeting their needs should be an integral part of a department development plan. The Key Stage 3 Strategy booklet Securing improvement: the role of subject leaders identifies three core roles for subject leaders in securing the progress of pupils. These are:

- judging standards;
- evaluating teaching and learning; and
- leading sustainable improvement.

Part of the role of a head of mathematics is to ensure that there is an effective learning environment across the department – one which promotes an ethos where pupils learning EAL feel secure in their learning and know that their contributions are respected.

About this guidance

This guidance is for mathematics teachers, heads of mathematics and EMA teachers in secondary schools. It is intended to help teachers support pupils learning EAL in the classroom in order to raise their attainment in mathematics lessons.

The guidance has five sections:

1. Pupils learning English: some considerations page 4
2. Speaking and listening in mathematics page 5
3. Reading and writing in mathematics page 8
4. Supporting teaching and learning in mathematics page 15
5. Activities for the mathematics classroom page 19

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In this booklet EMA advisers, EMA teachers and mathematics teachers describe how they have supported pupils learning EAL in mathematics classrooms.

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References

Framework for teaching mathematics: Years 7, 8 and 9 (DfES, 2001; ref. DfES 0020/2001)
www.standards.dfes.gov.uk/keystage3

Managing support for the attainment of pupils from minority ethnic groups (Ofsted, 2001)
www.ofsted.gov.uk

Mathematics and the use of language (SCAA, 1997; no longer available)

Securing improvement: the role of subject leaders (DfES, 2002; ref. DfES 0102/2002)
www.standards.dfes.gov.uk/keystage3

SMILE resources are available from SMILE Mathematics
www.smilemathematics.co.uk
Pupils learning English: some considerations

Pupils for whom English is an additional language are not a homogenous group. Extra planning and support may be required to take their specific learning needs into account. Pupils in your classes will vary in their experience of learning English.

In your school, pupils learning EAL may be in mathematics classes where:

- the majority of pupils share a common home language and cultural identity other than English – this is common in many inner-city schools;
- there are just one or two other pupils with a shared home language in common in the class;
- they are the only pupil learning EAL in the class or a speaker of a language not represented elsewhere in the school – an ‘isolated learner’.

The rate at which individual pupils learning EAL make progress in mathematics classrooms is likely to be determined by their literacy and schooling in their first language and their prior experiences of learning.

Mathematics teachers may be allocated some additional support for pupils learning EAL in their classes. The provision and frequency of staffing support will vary from school to school. Some possible ways of using the support effectively are listed below.

- Joint planning: where two teachers work together with a clearly defined, shared and negotiated role in all parts of the lesson.
- Joint teaching: where teachers can alternate in the roles of leading whole-class sessions or providing activities for small groups, pairs or individual pupils within lessons.
- Support staff acting as a mediator between the teacher and the pupils, to ensure that pupils learning EAL access the learning by clearly signposting activities, offering alternative explanations and assigning appropriate roles to pupils.
- Other adults may also offer advice on planning by: annotating planning, preparing additional resources, or suggesting appropriate teaching interventions.
- Support staff can provide feedback by sharing information on pupils’ needs within the class or a group.

A school's population can change over time. Which of the descriptions above most closely fits the situation in your school? Is it the same in all classes across Years 7, 8 and 9?

What support is available to your department in Key Stage 3? How is this used effectively?
Speaking and listening in mathematics

A focus on speaking and listening through oral and mental work in mathematics makes lessons more accessible to pupils for whom English is an additional language.

Skills in speaking and listening in mathematics are encouraged by the Key Stage 3 Strategy. Skills are developed by teachers and pupils exploring mathematical concepts through whole-class and group discussions in which teachers:

- question pupils effectively;
- give them time to think;
- expect them to demonstrate and explain their reasoning;
- explore reasons for any wrong answers.

Regular oral and mental work develops and secures pupils’ visualisation, thinking and communication skills. Talk enables learners to order and re-order their thinking. This is especially important for pupils for whom English is an additional language.

The following points are drawn from the leaflet Mathematics and the use of language (SCAA, 1997).

Effective mathematics teaching develops skills in speaking by:

- exploring mathematical concepts;
- describing visualisations of shapes, movements and constructions;
- explaining calculation strategies and talking about methods for the solution of problems;
- reasoning in working towards a solution and justifying results;
- comparing different mathematical processes for their efficiency and effectiveness;
- talking about mathematical expressions using mathematical and non-mathematical language;
- discussing which mathematical equipment and materials to use;
- comparing different solutions in order to arrive at a correct solution;
- discussing and interpreting data and drawing conclusions;
- presenting their findings to an audience.
Effective mathematics teaching develops skills in listening through:

- explanations from the teacher;
- non-mathematical explanations of mathematical ideas;
- mathematical language in order to acquire the vocabulary of mathematics;
- the teacher and other pupils exploring mathematical processes, reasoning and proving the solutions to a problem.

The Strategy's focus on direct interactive teaching encourages pupils to develop their listening skills. Effective teachers balance different teaching strategies by:

- giving well-structured demonstrations - for example, showing how to solve an algebraic equation or to interpret a graph;
- modelling mathematics and mathematical language using appropriate resources and visual displays;
- explaining and illustrating examples, referring to previous work or methods;
- questioning in ways that ensure all pupils can take part with appropriate support;
- summarising and reminding pupils of what has been taught and picking out key points and ideas;
- expecting pupils to use correct mathematical terms and notation and to talk about their insights rather than give single-word answers;
- using every opportunity to draw attention to new words or symbols with the whole class, in a group or when talking to individual pupils;
- ensuring that, as well as introducing new vocabulary, they consolidate familiar terms;
- being aware of the language demands of particular tasks and how words are used in a mathematics lesson.

Mathematics uses a range of vocabulary. In its technical vocabulary words have only a mathematical meaning – for example, algebra or trigonometry. In its specialist vocabulary words have specific meanings in mathematics that are different from their ordinary use – for example, mean, quarters, power or root; pupils need opportunities to practise both the mathematical and general meaning of such words. Grammatical words such as comparatives (less than), conditionals (if ... then) or connectives (unless) are also difficult and need practice.
Vasant Mahandru explains how he uses oral and mental starters to support Turkish-speaking bilingual pupils.

Oral and mental starters are an effective way of developing pupils’ mathematical language. The interactive nature of starters at the beginning of the lesson helps to develop pupils’ oracy skills and can also help them to understand sentence structures used in word problems.

One of the oral and mental starters we use in a Year 7 class is a ‘follow-me’ or loop card game aimed at enabling pupils to calculate using time. Each pupil has a card with a 24-hour clock time and a question.

**The time is 16:10.**

What was the time 35 minutes ago?

The first pupil reads out their card: ‘The time is 16:10. What was the time 35 minutes ago?’ The pupil with the answer says: ‘The time is 15:35 …’ and then reads out their question.

It is also important for pupils, bilingual learners particularly, to learn mathematical definitions. We used another loop card game to help pupils to reinforce these definitions. In this exercise each card includes a mathematical term and a definition for another term. On these cards Turkish translations for the answers were given to help the Turkish-speaking bilingual pupils.

**radius (yari çap)**

What do you call a quadrilateral with four equal sides but unequal angles?

**rhombus (eskanar dörtgen)**

What do you call a triangle with three equal sides?

Vasant Mahandru described one way he encourages the development of pupils’ speaking and listening skills.

- What activities do you use to develop these skills?
- Discuss ways in which you could strengthen speaking and listening skills and agree on at least three approaches that could be developed in mathematics lessons in your school.
Whole-class work provides opportunities for pupils to read texts, questions and problems aloud together, and written activities consolidate learning in mathematics.

The following points are drawn from the leaflet Mathematics and the use of language (SCAA, 1997).

Effective mathematics teaching develops skills in reading:

- recognising terminology, numbers, mathematical symbols and the technical language of mathematics;
- recognising patterns and relationships;
- reading mathematical expressions, formulae and problems;
- reading charts, diagrams, tables and graphs, in order to interpret data.

It develops skills in writing:

- using numbers and mathematical symbols and linking these in expressions, formulae and identities;
- using a variety of forms of mathematical presentation;
- redrafting presented information;
- setting out a mathematical argument or justification;
- formulating conclusions;
- explaining interpretations of graphs, charts or tables of data.

Reading and writing in mathematics is often quite different from reading and writing in other areas of the curriculum. Reading in mathematics:

- uses a wide range of text types and conventions – instructions, exercises and explanations, and a variety of tables, diagrams and charts;
- is not always linear – pupils are expected to refer backwards and forwards to information in tables and examples and to interrupt their reading while carrying out calculations.

Effective teachers model the reading of different types of mathematical texts and ensure that pupils are aware of how to access these different text types.

Confusions can arise when:

- similar combinations of elements do not always imply identical operations – for example, 45 is forty plus five, whereas 4a is four multiplied by a;
- the spatial arrangement of numbers and symbols carries different meanings – for example, $3^2$ is not the same as 35;
- symbols have different uses – for example, $48^\circ$ (the angle) means something different from $18^\circ$C (the temperature).

In mathematics, non-narrative writing uses several different forms:

- number statements linking numbers and mathematical symbols and occasionally a few words;
- a few sentences of prose as a response to questions like ‘why?’ ‘how?’ or ‘what do you notice?’ or in response to instructions such as ‘explain’ or ‘give reasons for your choice’;
diagrams, which might be geometrical, statistical or graphical;
- algebraic expressions;
- extended writing to describe strategies, reasoning and other mathematical processes, particularly in investigations or problem solving.

Effective teachers model the different forms of writing required. They ensure that pupils learning EAL are aware of the conventions used.

Catharine Driver and Karen Thomas describe how they developed pupils’ writing in mathematics.

Parliament Hill is a girls’ comprehensive school in north London. Around 30% of pupils are bilingual. Pupils from diverse ethnic backgrounds speak 44 different languages.

Staff analysed data on the achievement of different ethnic minority groups at the end of Key Stage 3. They identified a gap between the achievement of Bangladeshi girls and all other pupils in mathematics.

Data analysis suggested:
- more pupils of Bangladeshi heritage achieve level 5 on teacher assessment than in the national tests (this is true of test results over five years);
- the gap between the attainment of pupils of Bangladeshi heritage and all pupils is wider in mathematics (and science tests) than in English tests;
- the gap between the attainment of pupils of Bangladeshi heritage and all pupils is not as wide in teacher assessment as it is in the national tests.

The EAL coordinator, working with the LEA adviser, hypothesised that in oral work Bangladeshi pupils were achieving similar standards to their peers, but in test questions they were still having difficulties with literacy in mathematics and therefore scoring lower marks. Bangladeshi and other pupils learning EAL were still unfamiliar with the more formal language used in test questions.

EAL staff decided to work in partnership with the mathematics department to extend pupils’ use of language. The LEA adviser, the EAL coordinator and the head of mathematics met to discuss how they would develop pupils’ skills in writing about mathematical ideas. A training session was planned with the mathematics department. This included an introduction to the language features of mathematics and time to plan and prepare a model lesson for Year 7 pupils using a prompt sheet to focus on writing about mathematics. The teaching sequence is outlined overleaf.

The evaluation of this work highlighted a number of benefits.
- Mathematics teachers had devised similar lessons before but the involvement of the language specialist helped them to focus much more clearly on the language as well as the mathematics.
- The model teaching sequences went well, especially where they were team-taught by the EAL specialist and the mathematics teacher. Lessons proved to be beneficial to all pupils, not just the Bangladeshi girls.
- Pupils were fully involved in developing the lesson summary notes and this encouraged them to use the writing prompts in subsequent written work.
- Mathematics teachers have introduced the focus on writing lesson summary notes as a regular feature in their teaching.
Developing pupils’ skills in writing about mathematics

- a teaching sequence

**Familiarisation**
Look at examples of mathematics facts

**Identifying features**
Underline and label definitions, rules, examples, formulae

**Teacher modelling**
Writing on an OHT/board with contributions

**Guided writing**
Pupils writing with prompt sheet

**Independent writing**
Ready to try it alone!

Finding the area of a right-angled triangle

A right-angled triangle has one angle of 90°. The area is the amount of space inside a shape.

To find the area
- Draw another triangle to make a rectangle.
- Work out the area of the rectangle by multiplying the base by the height (or by counting the number of squares).
- Halve the answer because the area of the triangle is half the area of the rectangle.

Rule
The area of a right-angled triangle is the base multiplied by the height then halved or half of (base × height) or $\frac{\text{base} \times \text{height}}{2}$

Example
Base × height = 3 cm × 4 cm = 12 cm²
Half of 12 cm² = 12 cm² ÷ 2 = 6 cm²
The area of the triangle is 6 cm²

Remember to halve the area of the rectangle.
Fiction is rarely used in mathematics classrooms but a well-chosen text – for example, the story of the towers of Brahma or ‘One grain of rice’ by Demi – can be a useful starting point to contextualise mathematical problems. It can encourage pupils learning EAL to develop language within mathematics. It also allows planned opportunities for pupils to speak in their first language in classroom contexts.

Dice, board and number games, and ICT software also give opportunities to use language within a mathematical context.

Applying mathematics and solving problems provide good opportunities to develop reading and writing skills. Problems interest and challenge pupils to think hard about mathematics. They should be presented in straightforward English or the pupil’s home language, clearly set out, using visual clues, to help pupils understand the context. Words and phrases commonly used in word problems need to be taught systematically. This can be reinforced by asking pupils to write their own word problems.

5.67 + 3.38

These examples of word problems were written by pupils for whom English is an additional language. The problems were intended to interest young children.

\[
\begin{align*}
5.67 + 3.38 \\
\text{An elephant is about 5.67 metres high and its baby is 3.38 metres high. How much metres are they all together?}
\end{align*}
\]

\[
\begin{align*}
5.67 + 3.38 \\
\text{pinchios nose was 5.67 metres the 3rd time he lied, the next time he lied it growed 3.38 cm metres how long is his nose.}
\end{align*}
\]
A writing frame is useful to guide pupils in solving problems. They can use it to record their questions and strategies. Teachers encourage pupils to:

- work in pairs to discuss the problem and estimate an answer;
- highlight the relevant information;
- look words up in a dictionary if necessary;
- ask questions that show what they do not understand;
- break the calculation or solution to the equation into simpler steps;
- choose an appropriate and efficient method;
- check their work.

When appropriate, problems can be answered in pupils’ home languages.

**Using a writing frame**

**Read the problem.**

Two prime numbers are added. The answer is 45. What are the numbers?

*Underline key words*

**What is the relevant information?**

- I need to know the only even prime number is two.
- It must be something odd and add to 45.

**What calculations do I need to do?**

\[
2 + \square = 45 \\
2 + 43 = 45
\]

Is 43 a prime number?

Does anything go into this?

**Write the answer**

The two prime numbers are 43 and 2.

**Check your work**

\[
43 + 2 = 45 \text{ and I know they are both prime numbers.}
\]

In Key Stage 3 a strong emphasis is given to interpreting data, making inferences and drawing conclusions. Effective teachers model the interpretation of graphs, charts and tables and encourage pupils learning EAL to ask their own questions about the data represented. The teacher’s role in teasing out pupils’ understanding helps to develop pupils’ skills in interpreting data.
Samples of mathematical text from a variety of cultures can be used to illustrate work for pupils learning EAL. This is also useful for pupils whose home language is English. Pupils can be asked to work out what the problem is about and if possible solve it.

Can you solve this problem? What extra support do you need?

Z místa A vyjel do místa B cyklista průměrnou rychlostí 20 km/h. Za 45 minut vyjel z A do B motocyklista průměrnou rychlostí 44 km/h. Oba do místa B dojeli současně. Určete vzdálenost AB.
(The answer is 27.5 kilometres.)
What are the language demands in the following word problems? What additional support would you provide for pupils for whom English is an additional language? How would you expect pupils to solve the problems?

- The product of two numbers is 999. Their difference is 10. What are the two numbers?
- A shop was selling identical T-shirts. On Monday the shop offered 15% off the original price. On Friday the shop offered 20% off the original price. The price on Monday was 35p more than it was on Friday. What was the original price of the T-shirt?

Although nowadays most countries use the international set of numerals, a useful ‘maths club’ activity is using different number scripts, for example Bengali, Chinese or Punjabi, to create puzzles of addition, subtraction and multiplication squares, which other pupils can be asked to complete. Jigsaws using different number scripts are available commercially. Activities such as these acknowledge and value pupils’ cultural backgrounds.
These are some suggestions for making sure that teaching supports pupils learning English as an additional language.

**Before the lesson**

- Whenever possible plan jointly with EMA staff.
- Plan effective opportunities for talk to support mathematics learning.
- Plan the use of adult and pupil models of spoken English.
- Plan time for careful listening, oral exchange and supportive shared repetition.
- Prepare questions so that there is a balance of open and closed questions.
- Plan which pupils will be targeted with particular questions.
- Provide prompt cards and visual cues to support understanding.
- When appropriate, simplify the words but not the mathematics.
- Identify the mathematical vocabulary related to the unit of work that is being planned.
- Display vocabulary that is appropriate for the lesson.
- Plan the introduction of new words in a suitable context.

**During oral and mental starters**

- Encourage pupils learning EAL to join in when pupils are doing things in unison, for example, counting along a decimal number line or extending a number sequence.
- Build in thinking time.
- Ask pupils to discuss something with a partner; encourage them to practise responses in pairs, using home or first languages where appropriate; allow pupils time to rehearse any feedback to another pupil or an adult assistant.
Ask pupils to explain how they carried out a calculation, or solved an equation, or …

Make use of a variety of resources such as number fans, whiteboards and counting sticks to involve pupils learning EAL, allowing them to show rather than say their answers.

Be explicit about using specialist vocabulary and ensure that pupils also use the mathematical language modelled by the teacher.

Explain the meanings of new words carefully and rehearse them several times; encourage their use in context in oral sessions using focused questions.

Support pupils’ understanding by writing sentences on the board or using prompt cards.

During main teaching activities

- Model answers to questions.
- Ask pupils to offer their methods and solutions to the whole class for discussion.
- Encourage pupils to answer in complete sentences.
- Ensure that pupils learning EAL are supported effectively by an EMA teacher or teaching assistant.
- Display worked examples on OHT, board or flipchart so that pupils can follow a structure to support their explanations.
- Ensure pupils learning EAL have the opportunity to work collaboratively in a range of group and paired contexts:
  - with a pupil who shares the same first language when that is possible;
  - with pupils who provide good mathematical models as well as with pupils who provide good language models.
- Encourage discussion and cooperation between pupils.
- Build in time for pupils learning EAL to consolidate their use of mathematical vocabulary.
- Encourage pupils to use dual-language dictionaries and to list their own definitions.
- Encourage pupils to share their mathematical knowledge and experience from a range of cultural and linguistic perspectives.
During plenaries

- Summarise key facts and ideas.
- Highlight mathematical vocabulary.
- Invite pupils to present their work to the class and to talk about their strategies and conclusions.
- Encourage other pupils to ask questions and to reiterate what has been said in their own words.
- Go through a written exercise pupils did individually or in pairs to assess it informally and rectify any misconceptions or errors.
- Support pupils learning EAL in recording a summary of what they have learned.
I Identify which of the suggestions for oral and mental starters, for main teaching activities and for plenaries are already strong features of teaching mathematics to pupils learning EAL in your school.
I Identify which suggestions you would like to develop further in your teaching.
I Prioritise these suggestions and agree how you will put them into action.
Activities for the mathematics classroom

The following selection of activities to develop language in mathematics has been suggested by teachers. They can be used for oral and mental starters or as more extended activities during the main part of the lesson.

**How many things can you think of that …?**

Ask groups of pupils to list as many items as they can to match a particular definition. For example:
- How many things can you think of that ...
- ... have parallel lines?
- ... are cylindrical?
- ... have an even chance of occurring?
- ... are associated with the number ‘three’?

**‘Three in a row’ game**

- Give pairs of pupils, or ask them to make, a 3 by 3 matrix with cells labelled to link with a particular objective, such as:
  - Recognise and use multiples, factors (divisors), common factors, highest common factors, lowest common multiples and primes; use squares, positive and negative square roots, cubes and cube roots.

<table>
<thead>
<tr>
<th>Square number</th>
<th>Multiple of 7</th>
<th>Cube number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor of 24</td>
<td>Prime number</td>
<td>Triangular number</td>
</tr>
<tr>
<td>Negative number</td>
<td>Multiple of 4</td>
<td>Factor of 40</td>
</tr>
</tbody>
</table>

- Draw numbers from a pack of 1–100 number cards and call them out. Pairs try to put the numbers into the correct category. The first pair to get three in a row are the winners.

Objectives related to other aspects of the curriculum can be used. For example, in shape and space:
- Begin to identify and use angle, side and symmetry properties of triangles and quadrilaterals.
- Cells can be labelled with properties such as ‘two equal sides’, ‘a right angle’, ‘all angles equal’, ‘two pairs of parallel sides’, ‘two lines of symmetry’, ... and names of shapes called out.
- ‘Three in a row’ games can also be created for equivalence relationships between fractions, decimals and percentages, or between measures such as 3 hours 20 minutes, 200 minutes or 1200 seconds.
True or false?

Show and read statements written on cards. Ask pupils to show true/false cards to identify whether the statement is correct or incorrect. If the statement is false, ask a pupil to give a correct statement.

For example:

- **The radius is the distance around a circle.**
  - False
- **0.333333... = \(\frac{1}{4}\)**
  - False
- **The circumference is the distance around a circle.**
  - **True**
- **0.333333... = \(\frac{1}{3}\)**
  - **True**
- **The radius is a line joining the centre to a point on the circle.**
  - False
- **0.25 = \(\frac{1}{4}\)**
  - **True**

Jumbled sentences

Give pupils some jumbled sentences, for example:

- 160 is 60 of three-eighths.
- The angle 360 of sum a is degrees quadrilateral.

Ask pupils to reorganise the sentences so that they make mathematical sense.

Back to back

Give pupils geometrical drawings on cards. One pupil describes the drawing; the other draws it from the description.

This works well with tangram pieces arranged in a ‘picture’. One pupil has the picture on a card; the other has to assemble it.

How will you adapt these activities to use in your classrooms?

Do you have any other ideas for activities?